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List of the main results in the paper

MAXIMUM SIZE OF GRAPHS WITH GIRTH NOT LESS THAN A GIVEN NUMBER

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For graph G let $e(G)$ be the number of edges in G (the size of G), and let $\delta(G)$ and $\Delta(G)$ be the minimal degree and maximal degree of G respectively.

For $g \ge 3$ let $e_g(p)$ be the maximal size of a graph of order p with girth at least g.

Theorem 1. *For positive integer p we have*

$$e_5(p) \le \frac{1}{2}p\sqrt{p-1}.$$

Theorem 2. *If G is a simple graph of order p with girth ≥ 7 and $\delta(G) = \delta \ge 1$, then*

$$e(G) \le \frac{p}{4}\Big\{\frac{\delta^2-1}{2\delta-1} + \frac{1}{2\delta-1}\sqrt{(\delta^2-1)^2+4(2\delta-1)(p-1)}\Big\}.$$

Theorem 3. *Let G be a simple graph of order p with girth ≥ 7, $x = 2e(G)/p$, $\Delta(G) = \Delta$ and $\delta(G) = \delta \ge 1$. Then*

$$2mx^3 - x^2 + x - (p-1) \le 0,$$

where $m = \delta\Delta/(\Delta^2+\delta^2)$.

Corollary 4. *Let G be a simple graph of order p with girth ≥ 7, $\delta(G) = \delta \ge 1$ and $\Delta(G) = \Delta$. Then*

$$e(G) < \big(\frac{p}{2}\big)^{\frac{4}{3}}\sqrt[3]{\big(1+\frac{1}{\delta}\big)\big(\frac{\Delta}{\delta}+\frac{\delta}{\Delta}\big)}.$$

Theorem 4. *For $r \ge 2$ we have*

$$e_{2r+1}(p) < \frac{r}{r+1}p^{1+\frac{1}{r}} + p.$$

Corollary 5. *For $p \ge 3$ and $g > 3$ we have*

$$e_g(p) \le \frac{p}{p-2}e_g(p-1).$$